Weaving Mathematical Connections from Counting to Calculus:

Knowledge Clusters and The Quantile® Framework for Mathematics

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"Swiss cheese mathematics." This metaphor aptly describes some students' cognitive mathematical landscapes. When students miss connections among mathematical ideas, they may seem to have some substantive grasp and intuition on a topic at hand, yet they may still have major holes in their knowledge which prevent mastery of interrelated concepts. Those holes become limitations to the development of further mathematical knowledge.

In part, the Common Core State Standards for Mathematics (CCSSM), through a focus on coherence, help overcome the likelihood that these holes will emerge (NGA Center and CCSSO, 2010). The CCSSM incorporate coherence by stressing conceptual understanding of key ideas and continually returning to organizing principles such as place value or the properties of operations. The CCSSM align content standards across the grades so material learned in one grade explicitly builds upon concepts learned in previous grades. The introduction to the mathematics standards states,

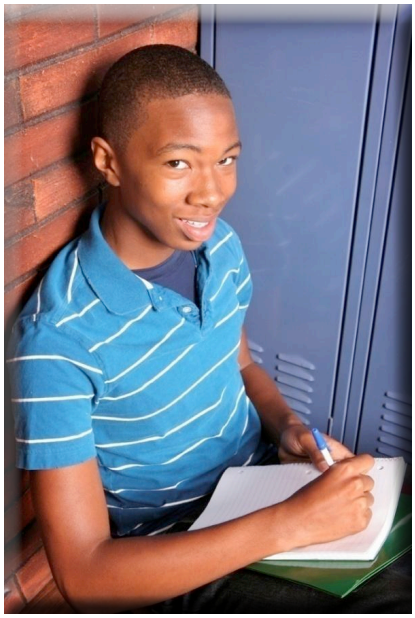
> ...the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is "a mile wide and an inch deep" (p. 3).

Learning Progressions & Coherence

Heritage (2008) explains that traditional sets of standards tend toward curricula that are "a mile wide and an inch deep" because they do not have the level of vertical articulation adopted by the CCSSM. Also, many state curricular frameworks specify objectives for each grade level but do not relay big ideas. Consequently, when objectives are taught, they tend to be given equal weight in terms of instructional time, even though their relative importance in terms of mathematical concepts is not equal. Heritage explains the usefulness of a learning progression to provide educators with a coherent sequencing of a curriculum which "helps teachers see connections between what comes before and after a specific learning goal, both in the short and long term" (p. 5).

The CCSSM are carefully constructed so mathematical concepts build on each other in a coherent fashion from one grade to the next and provide emphasis on "big ideas." The Quantile® Framework for Mathematics also was carefully constructed with a focus on vertical alignment and augments the CCSSM by providing a unique way to target students with resources that support their instructional needs. The Quantile Framework delineates almost five-hundred skills and concepts, named QTaxons, aligns the QTaxons with the CCSSM[1], and explicitly connect them together to form a tightly woven mathematical fabric.

Mathematical knowledge is hierarchical in nature and, possibly more readily than many subjects, lends itself to building learning progressions because the development of new concepts depends on the student's understanding of prerequisite ones. For instance, the concept of proportion depends upon first understanding fractions. To adequately understand fractions, one must have a firm grounding in multiplication and division. A strong foundation that connects and enriches conceptual understanding in mathematics enables students to meet future mathematical demands.



The Quantile Framework, with its QTaxons, explicitly connects concepts by detailing this hierarchy from kindergarten through high school, forming a web of prerequisite, supplemental, and impending skills and concepts in a vertically aligned taxonomy. Additionally, these QTaxons are associated with a difficulty measure so educators know the relative cognitive difficulty of one concept compared to another.

Schmidt et al (2002) have said that content standards and curricula are coherent if they are:

> …articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature of the disciplinary content from which the subject matter derives. That is, what and how students are taught should reflect not only the topics that fall within a certain academic discipline, but also the key ideas that determine how knowledge is organized and generated within that discipline (p. 9).

Used in conjunction, the Quantile Framework and the CCSSM empower educators as never before by enabling educators to target student ability and implement a coherent, vertically articulated mathematics learning progression from kindergarten through high school.

Heritage (2008) also acknowledges that students learn at different rates. Just as students do not grow at the same rate physically, not all students progress at the same rate in terms of their mathematical understanding. Some students may have a more superficial understanding of concepts because they are not cognitively ready to handle the complexities of more advanced mathematical ideas. Other students may have a much deeper understanding without much formal instruction. Consequently, "the spread of achievement increases with age," (p. 4).

Although age is a factor, it does not accurately pinpoint cognitive ability. Ultimately, it is up to educators to examine a student's ability in order to design instruction accordingly. To assist educators in this task, assessments linked with the Quantile Framework and ones that report in the Quantile metric provide a student Quantile measure[2]. In addition, the Quantile Framework provides Quantile measures for the QTaxons. By measuring both student's ability and concept difficulty on the same scale, educators can better target instruction to meet student needs.

[1] The Quantile website (http://www.Quantiles.com) provides two free tools which align the Quantile Framework with sets of standards including the CCSSM: The Math Skills Database and The Quantile Teacher Assistant.

[2] A student receives a Quantile measure by taking an assessment that is linked with the Quantile Framework or reports directly in the Quantile metric. Assessments which provide Quantile measures can be found on the Quantile website at http://www.quantiles.com/where-to-get-quantiles.aspx.

Knowledge Clusters

Within the Quantile Framework, each QTaxon is linked to related QTaxons forming a knowledge cluster. As described by Smith and Turner (2012) in "Policy Brief #4: A Mathematics Problem," "[t]hese knowledge clusters not only illustrate the interconnectivity of the skills and concepts, but also provide educators with actionable information they can use to target instruction, forecast understanding, and address student achievement."

Consider the essential concept of unit rate. Unit rate is an everyday skill used when comparing relative value. The Quantile Framework identifies the concept of unit rate and also identifies its relative difficulty to other concepts using a measurement scale. The framework explicitly describes this QTaxon as "calculate unit rates to make comparisons."

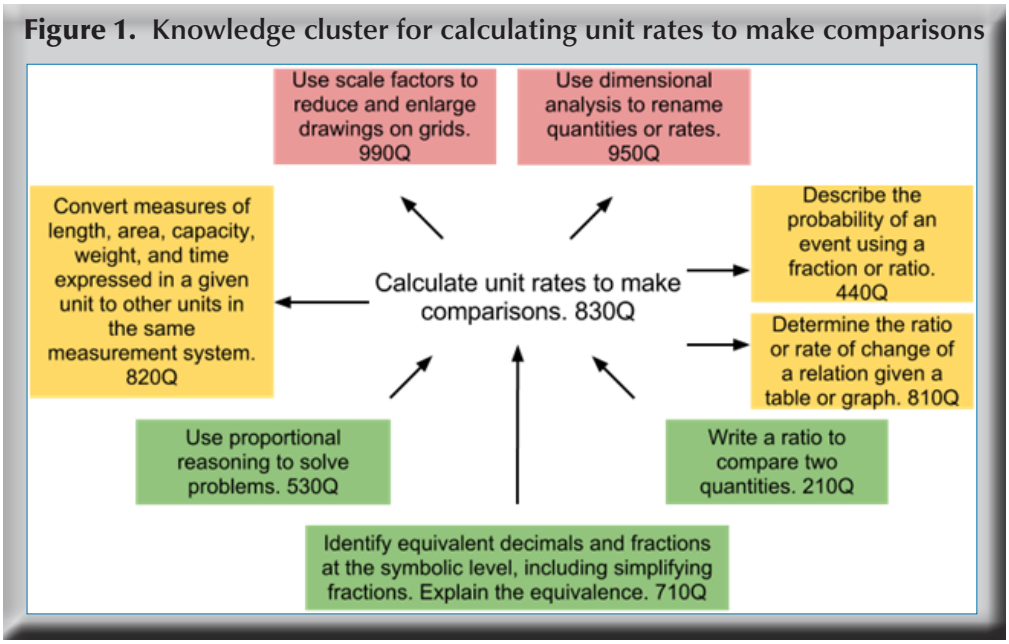
The knowledge cluster for calculating and comparing unit rates (Houghton, 2011) is shown in Figure 1. The QTaxons at the bottom of the figure, shown in green, are prerequisite skills and concepts a student needs in order to be ready to learn about calculating and comparing unit rates. The skills shown in red listed above the focus QTaxon are impending skills that a student should be prepared to learn after gaining an understanding of unit rates. QTaxons shown in yellow are classified as supplemental QTaxons because they identify skills related to the focus QTaxon and may enrich instruction with other applications or provide opportunities for differentiation. The difficulty value associated with each QTaxon description is also shown. This value is a Quantile measure that describes the cognitive demand (difficulty) of that material at an introductory level.

With Quantile knowledge clusters, teachers are empowered in many ways. Given the Quantile measures of the QTaxons and the Quantile measure of a student, a teacher is able to target instruction by matching these two values. Matching the student measure with the Quantile measure of the QTaxon helps students to be successful with the material and continue to grow. When mathematics educators use the CCCSM in conjunction with the Quantile Framework, they have vertical integration of concepts and skills and they can identify which students are cognitively ready for instruction. If some students are not ready for instruction, educators have the information to gauge which skills students are ready for, as well as identify which skills can be utilized to help prepare students for learning the focus skill.

When students take end of unit tests or statewide assessments, results may not always directly inform teachers' instructional practices. These interim and summative assessments often result in a score that summarizes in a single value what the student learned. These scores typically provide information regarding accountability or overall student learning, but teachers often have difficulty using this data to shape instruction or help students improve their own learning (Black & William, 2004).

Assessment practices used to tailor instructional activities to the students' specific needs are called formative assessments because they help to "form" appropriate instruction or what Black and William call the "formative purpose" (2004). When a student takes a summative assessment that is linked with The Quantile Framework for Mathematics, the student will also receive a Quantile measure. Educators can then use these student Quantile measures along with the QTaxons and knowledge clusters for a formative purpose.

Figure 1. Knowledge cluster for calculating unit rates to make comparisons



Learning Frontiers

A student's learning frontier ranges from 50Q above to 50Q below his or her Quantile measure. So a student with a Quantile measure of 720Q has a learning frontier that ranges from 670Q to 770Q. That student is cognitively prepared to learn about equivalent decimals and fractions, a skill which has a measure of 710Q. Note that the student is also likely to easily manage the material of proportional reasoning to solve problems, which has a measure of 530Q. So a teacher using the Quantile Framework could help a student move toward "understanding calculating unit rates to make comparisons" due to the articulation and connectivity of the skills identified in the knowledge cluster. Appropriately designed tasks will provide students insights about unit rates, drawing on their proportional reasoning skills.

Teachers now can confidently identify students whose Quantile measures are above the learning frontier for calculating unit rates to make comparisons, (i.e. students whose measures exceed 880Q). Supplemental exercises could be provided for these students which help draw connections to the supplemental QTaxons of converting measures (820Q), examining rates of change in tables and graphs (810Q), and probabilities of events (440Q).

Conclusion

Through the process of making connections to related concepts, students deepen their mathematical understanding. In addition, boredom in the classroom is minimized by challenging students to deepen their knowledge and not just broaden it.

With the CCSSM emphasis on focus and coherence, mathematics teachers become "master weavers" as they teach their students to intertwine conceptual mathematical threads and create their own unique fabric of mathematics—their own mathematical cognitive knowledge structure. Students develop mathematical intuition and acumen which increases their ability to solve mathematical applications found outside of the classroom.

Educating all students in mathematics requires targeted and cohesive instruction. Designing and implementing targeted instruction is challenging, but necessary. Teachers need support in effectively integrating targeted instruction in their classroom. Together, the CCSSM and The Quantile Framework for Mathematics help educators meet this challenge by mapping vertical articulation and providing an estimate of the cognitive demand of concepts.

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